

LS-DYNA®'s Linear Solver Development — Phase 1: Element Validation

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- Introduction
- Line elements
- Surface elements
- Solid elements
- Summary

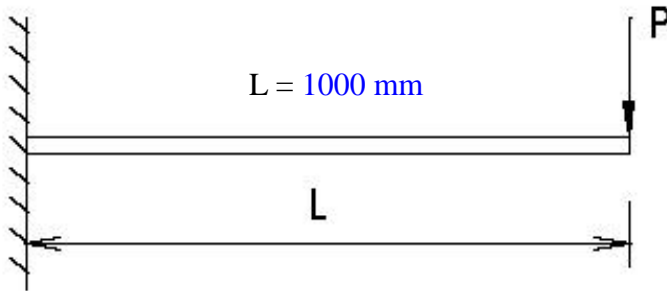
LSTC is taking a big effort to develop and improve the linear solution capabilities in LS-DYNA® due to the increasing request from users.

- This is a joint project between Ford and LSTC to validate the linear solvers in LS-DYNA.
- Validations between LS-DYNA and NASTRAN.
- The NASTRAN results are provided by Ford Motor Company.
- Identify the corresponding elements, material models, boundary conditions, loading types, and solution types in LS-DYNA which have the best match with the counterparts in NASTRAN.

Compared elements between LS-DYNA and NASTRAN in this paper.

- Line elements: CBAR, CBEAM, CBEAM3, CBEND, CROD, CONROD and CTUBE
- Shell elements: CTRIA3, CTRIA6, CQUAD4, and CQUAD8.
- Solid elements: CTETRA4, CTETRA6, CPENTA6, CPENTA15, CHEXA8 and CHEXA20

Objective – Modeling Validation



- Material – Steel
 - Elastic Modulus $E = 210000.0$ (MPa)
 - Poison Ratio $\nu = 0.3$
 - Density $\rho = 7.85E-09$ (Ton/mm³)
- Two types of analyses:
 - Static Analysis
 - with $P=100$ N Normal modes with Fixed at left end
 - First 10 Modes
- **Purpose:**
 - Validate **CBAR** with **PBAR** & **PBARL** equivalent in **LS-DYNA**
- Use a simple cantilever beam model to validate the response from two simple analyses to validate the response
 - Static analysis
 - Normal Mode
- Validate response data from Handbook, MSC/NASTRAN and/or Altair/OptiStruct.

Three cases are investigated for BAR element: PBARL, PBARn and PBARu.

PBARL entry is used to define properties of CBAR entry by cross-sectional dimensions

1	2	3	4	5	6	7	8	9	10
PBARL	PID	MID	GROUP	TYPE					
	DIM1	DIM2	DIM3	DIM4	DIM5	DIM6	DIM7	DIM8	
	DIM9	-etc.-	NSM						

- PBARn means using the calculated values of PBARL into PBAR cards.
- PBARu means using the values from user direct input to PBAR cards..

1	2	3	4	5	6	7	8	9	10
PBAR	PID	MID	A	I1	I2	J	NSM		
	C1	C2	D1	D2	E1	E2	F1	F2	
	K1	K2	I12						

LS-DYNA beam type 2 is used to compared with NASTRAN for PBARL case.

Card 2	1	2	3	4	5	6	7	8
Variable	STYPE	D1	D2	D3	D4	D5	D6	
Type	A10	F	F	F	F	F		

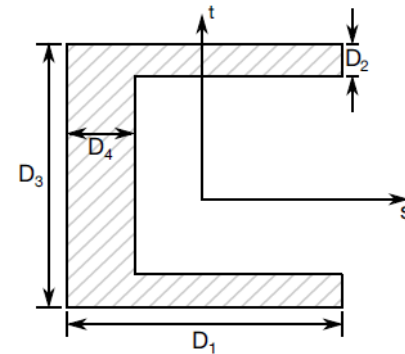
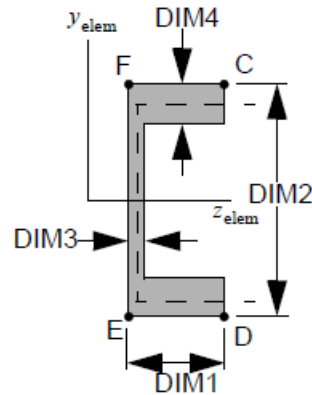
LS-DYNA beam type 13 is used to compared with NASTRAN for PBARn and PBARu cases.

Card 2	1	2	3	4	5	6	7	8
Variable	A	ISS	ITT	J	SA	IST		
Type	F	F	F	F	F	F		

Please note that LS-DYNA has only one parameter (SA) for area factor of shear, $SA = K1$ or $K2 * \text{Area}$. NASTRAN default is infinite for PBARu case, it can set a large number to match with NASTRAN.

The ISS and ITT value (area moments of inertia) in LS-DYNA should be reversed from NASTRAN I1, I2 value (area moments of inertia) due to the different local coordinate definition

- ROD
- TUBE
- TUBE2
- I
- CHAN
- T
- BOX
- BAR
- CROSS
- H
- T1
- I1
- CHAN1
- Z
- CHAN2
- T2
- BOX1
- HEXA
- HAT
- HAT1
- DBOX



NASRTAN CHAN

- DIM1
- DIM2
- DIM3
- DIM4

LS-DYNA Channel Section_02

- D1
- D3
- D4
- D2

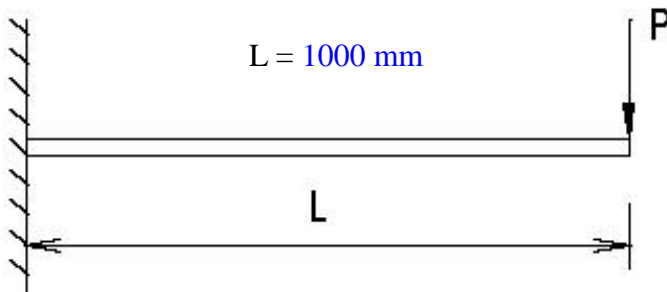
	PBARL			PBARn			PBARu		
	NASTRAN	LS-DYNA	Diff	NASTRAN	LS-DYNA	Diff	NASTRAN	LS-DYNA	Diff
Dz (mm)	-280.45	-281.29	-0.30%	-280.45	-280.47	-0.01%	-280.39	-280.39	0.00%
Mode # (Hz)									
1	9.20	9.19	0.16%	9.20	9.20	0.01%	9.20	9.20	0.00%
2	23.16	23.14	0.07%	23.16	23.16	0.00%	23.19	23.19	0.00%
3	57.61	57.56	0.10%	57.61	57.58	0.06%	57.67	57.66	0.02%
4	144.03	144.71	0.47%	144.03	144.01	0.01%	145.32	145.31	0.01%
5	161.09	161.11	0.01%	161.09	160.89	0.13%	161.48	161.43	0.03%
6	315.04	315.57	0.17%	315.04	314.35	0.22%	316.43	316.31	0.04%
7	398.44	403.81	1.35%	398.44	398.35	0.02%	406.91	406.82	0.02%
8	519.44	521.39	0.38%	519.44	517.69	0.34%	523.09	522.83	0.05%
9	767.50	778.39	1.42%	767.50	767.23	0.03%	781.40	780.93	0.06%
10	773.49	787.39	1.80%	773.49	769.78	0.48%	797.38	797.11	0.03%

NASTRAN	LS-DYNA	NASTRAN	LS-DYNA
01-ROD	08 Circular	12-I1	15 I-Shape3 01 I-Shape (symmetric)
02-TUBE	09 Tubular	13-CHAN1	16 Channel2
03-TUBE2	09 Tubular	14-Z	06 Z-Shape
04-I	10 I-Shape2 01 I-Shape (symmetric)	15-CHAN2	17 Channel3
05-CHAN	02 Channel	16-T2	18 T-Shape3
06-T	04 T-Shape	17-BOX1	19 Box-Shape 2
07-BOX	05 Box-Shape	18-HEXA	20 Hexagon
08-BAR	11 Solid Box	19-HAT	21 Hat Shape
09-CROSS	12 Cross	20-HAT1	22 Hat Shape 2(limited)
10-H	13 H-shape	21-DBOX	N/A
11-T1	14 T-Shape2	N/A	07 Trapezoidal-Shape

It can see that there is a good match between LS-DYNA and NASTRAN.

- PBARu case results are best matched as the input parameters are almost same.
- If the K1 is not equal to K2, it may affect results at some eigen frequencies for PBARn case.
- There is some difference in PBARL case due to the fact that input parameters are different in some shapes.

Objective – Modeling Validation



- Material – Steel

- Elastic Modulus $E = 210000.0$ (MPa)
- Poison Ratio $\nu = 0.3$
- Density $\rho = 7.85E-09$ (Ton/mm³)

- Purpose:

- Validate **CBEAM** with **PBEAM** & **PBEAML** equivalent in **LS-DYNA**
- Use a simple cantilever beam model to validate the response from two simple analyses to validate the response
 - Static analysis
 - Normal Mode
- Validate response data from Handbook, MSC/NASTRAN and/or Altair/OptiStruct.

Regarding the NASTRAN manual, one needs to use the CBEAM element instead of the CBAR element if any of the following features is important:

- The neutral axis and shear center do not coincide.
- The effect of cross-sectional warping on torsional stiffness is critical.
- The difference in the mass center of gravity and the shear center is significant.

LS-DYNA beam type 2 is used to compared with NASTRAN for PBEAML case.

Card 2	1	2	3	4	5	6	7	8
Variable	STYPE	D1	D2	D3	D4	D5	D6	
Type	A10	F	F	F	F	F		

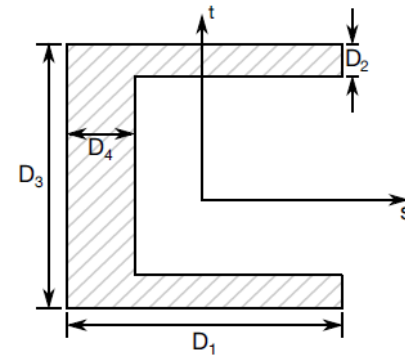
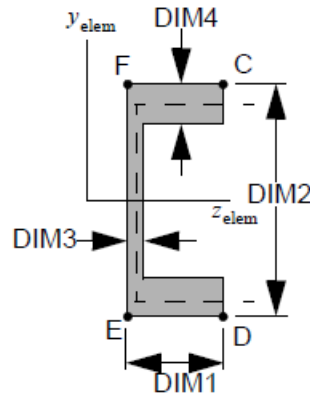
LS-DYNA beam type 13 is used to compared with NASTRAN for PBEAMn and PBEAMu cases.

Card 2	1	2	3	4	5	6	7	8
Variable	A	ISS	ITT	J	SA	IST		
Type	F	F	F	F	F	F		

Please note that LS-DYNA has only one parameter (SA) for area factor of shear, $SA = K1 \text{ or } K2 * \text{Area}$. The default value of area factor for shear (K1, K2) is 1.0 for PBEAMu case, it can use $1.0 * \text{Area}$ in LS-DYNA.

The ISS and ITT value (area moments of inertia) in LS-DYNA should be reversed from NASTRAN I1, I2 value (area moments of inertia) due to the different local coordinate definition.

- ROD
- TUBE
- TUBE2
- I
- CHAN
- T
- BOX
- BAR
- CROSS
- H
- T1
- I1
- CHAN1
- Z
- CHAN2
- T2
- BOX1
- HEXA
- HAT
- HAT1
- DBOX
- L



NASRTAN CHAN

- DIM1
- DIM2
- DIM3
- DIM4

LS-DYNA Channel Section_02)

- D1
- D3
- D4
- D2

	PBEAML			PBEAMn			PBEAMu		
	NASTRAN	LS-DYNA	Diff	NASTRAN	LS-DYNA	Diff	NASTRAN	LS-DYNA	Diff
Dz (mm)	-280.45	-281.29	-0.30%	-280.44	-280.47	-0.01%	-280.41	-280.42	0.00%
Mode # (Hz)									
1	9.20	9.19	0.16%	9.20	9.20	0.01%	9.20	9.20	0.02%
2	22.65	23.14	2.17%	22.65	23.16	2.24%	23.18	23.16	0.09%
3	57.61	57.56	0.10%	57.61	57.58	0.06%	57.65	57.58	0.11%
4	88.15	93.90	6.53%	88.15	90.68	2.88%	90.41	90.96	0.61%
5	136.96	144.71	5.66%	136.96	144.01	5.15%	144.93	144.01	0.64%
6	161.09	161.11	0.01%	161.09	160.89	0.13%	161.31	160.89	0.26%
7	253.31	281.68	11.20%	253.31	272.03	7.39%	271.25	272.86	0.59%
8	315.04	315.57	0.17%	315.05	314.35	0.22%	315.84	314.35	0.47%
9	348.11	403.81	16.00%	348.11	398.35	14.43%	404.31	398.35	1.47%
10	425.95	469.39	10.20%	425.95	453.31	6.42%	452.16	454.69	0.56%

NASTRAN	LS-DYNA	NASTRAN	LS-DYNA
01-ROD	08 Circular	12-I1	15 I-Shape3 01 I-Shape (symmetric)
02-TUBE	09 Tubular	13-CHAN1	16 Channel2
03-TUBE2	09 Tubular	14-Z	06 Z-Shape
04-I	10 I-Shape2 01 I-Shape (symmetric)	15-CHAN2	17 Channel3
05-CHAN	02 Channel	16-T2	18 T-Shape3
06-T	04 T-Shape	17-BOX1	19 Box-Shape 2
07-BOX	05 Box-Shape	18-HEXA	20 Hexagon
08-BAR	11 Solid Box	19-HAT	21 Hat Shape
09-CROSS	12 Cross	20-HAT1	22 Hat Shape 2(limited)
10-H	13 H-shape	21-DBOX	N/A
11-T1	14 T-Shape2	N/A	07 Trapezoidal-Shape
		22-L	03 L-shape

It can see that there is a good match between LS-DYNA and NASTRAN.

- PBARu case results are best matched as the input parameters are almost same.
- If the K1 is not equal to K2, it may affect results at some eigen frequencies for PBARN case.
- There is some difference in PBARL case due to the fact that input parameters are different in some shapes.
- The torsional modes have been just implemented, and still needs to improve the accuracy of the torsional modes calculation.

The CBEAM3 is a general three-node beam element that has been implemented as a curved one-dimensional Timoshenko beam element so that both the initial curvatures of beam reference axis and the cross-section shears are included in the formulation of the linear strain-displacement relations.

LS-DYNA does not have general three-node beam element at this time.

The bend element is defined with a CBEND entry and its properties are defined with a PBEND entry. The BEND element is a one-dimensional bending element with a constant radius of curvature. The CBEND element may be used to analyze either curved beams or pipe elbows. The bend element includes extension, torsion, bending in two perpendicular planes, and the associated transverse shear.

LS-DYNA beam element 14 is elbow (straight or curved pipe) integrated tubular beam element. The input parameters of bend element are different in two codes, it needs figure out the corresponding parameters in future for further validation.

The rod element is defined with a CROD entry and its properties with a PROD entry. The rod element includes extensional and torsional properties.

1	2	3	4	5	6	7	8	9	10
PROD	PID	MID	A	J	C	NSM			

The CONROD entry is an alternate form that includes both the connection and property information on a single entry.

1	2	3	4	5	6	7	8	9	10
CONROD	EID	G1	G2	MID	A	J	C	NSM	

The tube element is a specialized form that is assumed to have a circular cross section. The tube element is defined with a CTUBE entry, and its properties with a PTUBE entry

1	2	3	4	5	6	7	8	9	10
PTUBE	PID	MID	OD	T	NSM	OD2			

LS-DYNA beam element 3 (truss) can be used to compare with NASTRAN ROD element.

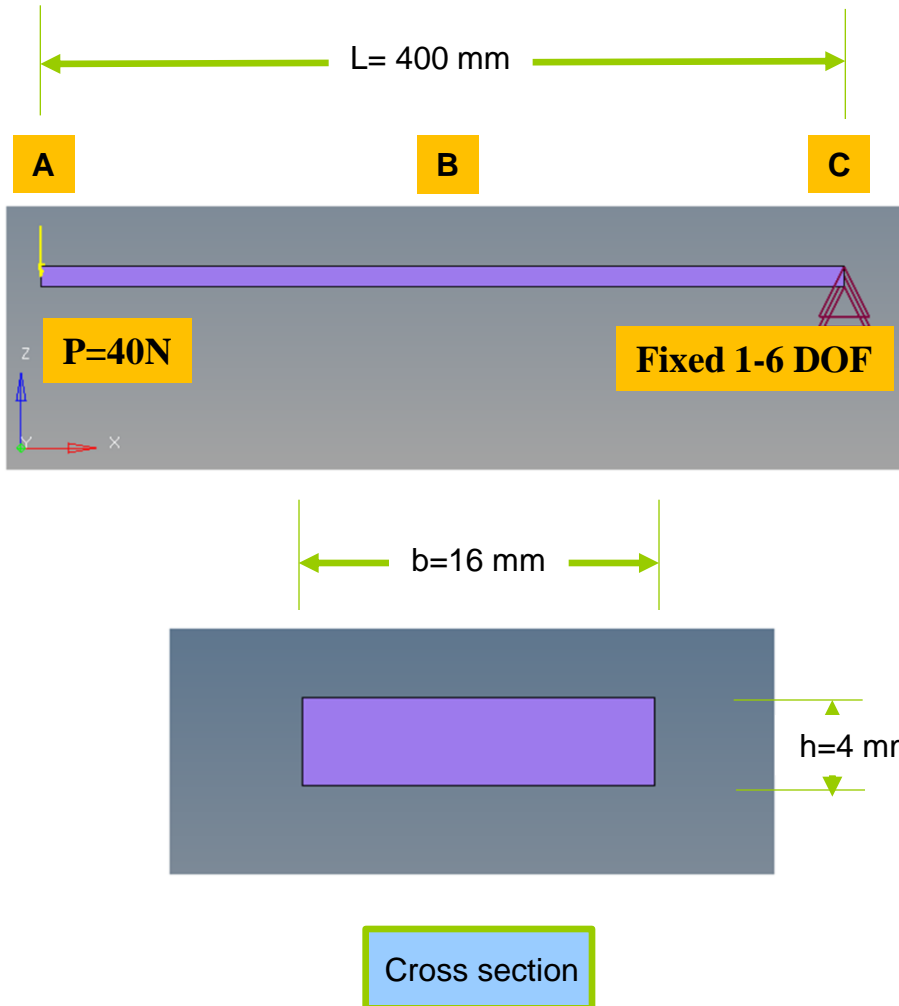
Card 2	1	2	3	4	5	6	7	8
Variable	A	RAMPT	STRESS					
Type	F	F	F					

Card 2	1	2	3	4	5	6	7	8
Variable	STYPE	D1	D2	D3	D4	D5	D6	
Type	A10	F	F	F	F	F		

	CROD			CONROD			CTUBE		
	NASTRAN	LS-DYNA	Diff	NASTRAN	LS-DYNA	Diff	NASTRAN	LS-DYNA	Diff
Dx (mm)	1.52E-03	1.52E-03	0.01%	1.52E-03	1.52E-03	0.01%	1.52E-03	1.52E-03	0.01%
Mode # (Hz)									
1	1293.05	1290.91	0.17%	1293.05	1290.91	0.17%	1293.05	1290.91	0.17%
2	3879.15	3873.05	0.16%	3879.15	3873.05	0.16%	3879.15	3873.05	0.16%
3	6465.24	6456.14	0.14%	6465.24	6456.14	0.14%	6465.24	6456.14	0.14%
4	9051.34	9040.81	0.12%	9051.34	9040.81	0.12%	9051.34	9040.81	0.12%
5	11637.43	11627.70	0.08%	11637.43	11627.70	0.08%	11637.43	11627.70	0.08%
6	14223.51	14217.44	0.04%	14223.51	14217.44	0.04%	14223.51	14217.44	0.04%
7	16809.57	16810.66	0.01%	16809.57	16810.66	0.01%	16809.57	16810.66	0.01%
8	19395.60	19408.00	0.06%	19395.60	19408.00	0.06%	19395.60	19408.00	0.06%
9	21981.59	22010.10	0.13%	21981.59	22010.10	0.13%	21981.59	22010.10	0.13%
10	24567.51	24617.59	0.20%	24567.51	24617.59	0.20%	24567.51	24617.59	0.20%

For the ROD elements, there is good match between LS-DYNA and NASTRAN.

Problem Statement



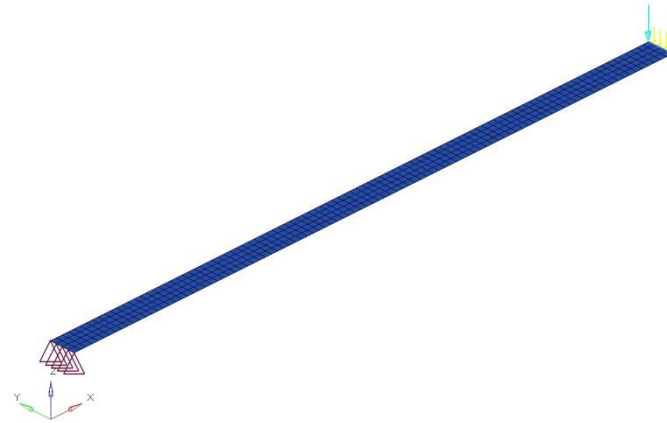
- Force $P = 40 \text{ N}$ is applied on the edge at the location A.
- Boundary are fixed in all directions at location C.
- Extract Displacement at location A
- Extract Stresses at Location B and C
- Material properties – Steel
 - $E = 210000 \text{ MPa}$
 - $\nu = 0.3$
 - $\rho = 7.91 \text{E-}09 \text{ Mg}$

TRIA and QUAD elements

CTRIA3 and CTRIA6



CQUAD4 and CQUAD8



NASTRAN	LS-DYNA
CTRIA3	Shell 18,20 and 21
CTRIA6	Shell 24
CQUAD4	Shell 18,20 and 21
CQUAD8	Shell 23

	CTRIA3			CTRIA6		
	NASTRAN	LS-DYNA 20	Diff	NASTRAN	LS-DYNA 24	Diff
Dz (mm)	-47.4624	-47.4617	-0.001%	-47.4849	-47.4777	-0.015%
Mode # (Hz)						
1	20.8669	20.7631	0.497%	20.8604	20.8623	0.009%
2	88.3231	88.4559	0.150%	83.2266	83.2339	0.009%
3	130.7269	130.0598	0.510%	130.6804	130.6977	0.013%
4	365.9040	363.9614	0.531%	365.7288	365.8046	0.021%
5	549.3415	550.1863	0.154%	517.7407	517.7508	0.002%
6	716.6975	712.6790	0.561%	716.2107	716.4403	0.032%
7	945.4971	911.7630	3.568%	928.0434	918.6217	1.015%
8	1184.1670	1177.0640	0.600%	1182.9780	1183.5570	0.049%
9	1520.0080	1522.4410	0.160%	1433.0080	1432.9120	0.007%
10	1767.8440	1756.4100	0.647%	1765.4180	1766.6230	0.068%

	CQUAD4			CQUAD8		
	NASTRAN	LS-DYNA 20	Diff	NASTRAN	LS-DYNA 23	Diff
Dz (mm)	-47.4769	-47.4736	-0.007%	-47.4835	-47.4771	-0.013%
Mode # (Hz)						
1	20.8626	20.8632	0.003%	20.8608	20.8623	0.007%
2	83.2402	83.5015	0.314%	83.2280	83.2343	0.008%
3	130.7130	130.7083	0.004%	130.6838	130.6923	0.007%
4	365.9359	365.8822	0.015%	365.7407	365.7740	0.009%
5	518.0620	519.6969	0.316%	517.7523	517.7132	0.008%
6	716.9801	716.7585	0.031%	716.2390	716.3465	0.015%
7	937.0877	912.2967	2.646%	924.0383	907.2819	1.813%
8	1185.0670	1184.4520	0.052%	1183.0350	1183.3200	0.024%
9	1435.0200	1439.5960	0.319%	1433.0410	1432.6690	0.026%
10	1770.0790	1768.7090	0.077%	1765.5210	1766.1620	0.036%

For the shell elements, there is good match between LS-DYNA and NASTRAN.



- CTETRA -- Four-sided solid element with 4 to 10 nodes
- CPENTA -- Five-sided solid element with 6 to 15 nodes
- CHEXA -- Six-sided solid element with 8 to 20 nodes.

NASTRAN	LS-DYNA
CTETRA4	Solid 10
CTETRA10	Solid 16 and 17
CPENTA6	Solid 15
CPENTA15	Solid 15
CHEXA8	Solid 18
CHEXA20	Solid 23

	CTETRA4			CTETRA10		
	NASTRAN	LS-DYNA 10	Diff	NASTRAN	LS-DYNA 16	Diff
Dz (mm)	-18.63804	-18.63800	0.000%	-48.17959	-48.17960	0.000%
Mode # (Hz)						
1	33.2550	33.5053	0.753%	20.6594	20.6791	0.096%
2	89.3804	90.0539	0.753%	82.0754	82.1535	0.095%
3	208.4084	209.9997	0.764%	129.4367	129.5605	0.096%
4	555.8498	560.0637	0.758%	362.2688	362.6156	0.096%
5	583.6141	588.0961	0.768%	510.9936	511.4776	0.095%
6	1139.2660	1148.0150	0.768%	709.4455	710.1253	0.096%
7	1537.8000	1549.4570	0.758%	902.8652	903.4024	0.059%
8	1572.4480	1579.5940	0.454%	1171.7590	1172.8830	0.096%
9	1891.6660	1906.3560	0.777%	1415.4800	1416.8100	0.094%
10	2814.6370	2837.2200	0.802%	1748.5480	1750.2260	0.096%

	CPENTA6			CPENTA15		
	NASTRAN	LS-DYNA 15	Diff	NASTRAN	LS-DYNA 15	Diff
Dz (mm)	-47.40669	-46.9242	1.018%	-48.47603	-46.9242	3.201%
Mode # (Hz)						
1	20.8828	21.1442	1.252%	20.5754	21.1442	2.764%
2	89.0364	89.5456	0.572%	82.1549	89.5456	8.996%
3	130.8285	132.4522	1.241%	128.9275	132.4522	2.734%
4	366.2072	370.6620	1.216%	360.8806	370.6620	2.710%
5	553.7140	556.9436	0.583%	511.4554	556.9436	8.894%
6	717.3573	725.3578	1.115%	706.7874	725.3578	2.627%
7	934.7210	1118.9560	19.710%	895.6508	1118.9560	24.932%
8	1185.3970	1201.3460	1.345%	1167.4580	1201.3460	2.903%
9	1531.8210	1540.9160	0.594%	1416.6730	1540.9160	8.770%
10	1769.9740	1791.8410	1.235%	1742.2190	1791.8410	2.848%

LS-DYNA solid type 15 does not specify the node number, so it is used to compare with both 6 and 15 nodes elements. It looks like the solid type 15 is 6 nodes element as the results of 6 nodes are better than 15 nodes.

	CHEXA8			CHEXA20		
	NASTRAN	LS-DYNA 18	Diff	NASTRAN	LS-DYNA 23	Diff
Dz (mm)	-47.41147	-47.40660	0.010%	-48.42445	-46.75420	3.449%
Mode # (Hz)						
1	20.8814	20.8824	0.005%	20.5899	21.2327	3.122%
2	83.3226	83.3214	0.001%	82.2044	84.3201	2.574%
3	130.8375	130.8438	0.005%	129.0079	133.0611	3.142%
4	366.3220	366.3418	0.005%	361.0798	372.5714	3.183%
5	518.5689	518.5616	0.001%	511.7409	524.8207	2.556%
6	717.8517	717.8992	0.007%	707.1247	730.0856	3.247%
7	926.1385	926.2946	0.017%	894.0481	995.3020	11.325%
8	1186.7640	1186.8630	0.008%	1167.9270	1206.8740	3.335%
9	1436.4120	1436.3930	0.001%	1417.4130	1453.7380	2.563%
10	1773.0960	1773.2830	0.011%	1742.7870	1802.8370	3.446%

The results of LS-DYNA solid type 18 much better than type 23 as solid 18 is based on linear theory.

- A good match between LS-DYNA and NASTRAN are reached for most of cases.
- Pointed out the area where LS-DYNA needs further development and improvement.
- Help users to get better understanding on the current status of LS-DYNA's linear solvers
- Provides suggestions of how to select the types of elements and parameters in LS-DYNA for translating finite element models between NASTRAN and LS-DYNA.

Thank you